Critical-Layer Enhancement of Mesoscale Eddy Stirring in the Southern Ocean

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ABSTRACT

Meridional cross-sections of effective diffusivity in the Southern Ocean are presented and discussed. The effective diffusivity, $K_{\text{eff}}$, characterizes the rate at which mesoscale eddies stir properties on interior isopycnal surfaces and laterally at the sea surface. The distributions are obtained by monitoring the rate at which eddies stir an idealized tracer whose initial distribution varies monotonically across the Antarctic Circumpolar Current (ACC). In the absence of observed maps of eddying currents in the interior ocean, the advecting velocity field is taken from a state estimate of the Southern Ocean (SOSE) in the presence of eddies. A three-dimensional advection-diffusion equation is solved and the diffusivity diagnosed by applying the Nakamura (1996) technique on both horizontal and isopycnal surfaces. The resulting meridional sections of $K_{\text{eff}}$ reveal regions of intensified isopycnal eddy stirring (reaching values of $\sim 2000 \text{ m}^2 \text{ s}^{-1}$) beneath, and on the equatorward flank of, the ACC. Lower effective diffusivity values ($\sim 500 \text{ m}^2 \text{ s}^{-1}$) are found near the surface where the mean flow of the ACC is strongest. It is argued that $K_{\text{eff}}$ is enhanced in the vicinity of the steering level of baroclinic waves, which is deep along the axis of the ACC but shallows on the equatorial flank. This pattern is referred to as a critical layer. Values of $K_{\text{eff}}$ are also found to be spatially correlated with gradients of potential vorticity on isopycnal surfaces and are large (small) where those gradients are weak (strong), as is to be expected from simple dynamical arguments. Finally, implications of the spatial distributions of $K_{\text{eff}}$ for the dynamics of the ACC and its overturning circulation are discussed.
1. Introduction

Here we infer rates of eddy mixing in the Southern Ocean by monitoring the evolution of a numerical tracer governed by an advection-diffusion equation. This work builds on the study of Marshall et al. (2006, henceforth MSJH) who drove an advection diffusion equation with surface velocities computed from satellite altimetry. In that study, an initial tracer distribution with a prescribed monotonic gradient across the Antarctic Circumpolar Current (ACC) was stirred and mixed by the two-dimensional eddying flow. The theoretical framework set out by Nakamura (1996) was then applied to the tracer distribution. The resulting “effective diffusivity,” $K_{eff}$, characterizes the rate of mixing by eddies acting laterally at the sea surface. An interesting meridional structure emerged, with enhanced mixing rates ($\sim 2000 \text{ m}^2 \text{ s}^{-1}$) on the equatorial flank and evidence of suppressed mixing ($\sim 500 \text{ m}^2 \text{ s}^{-1}$) near the core of the ACC.

Smith and Marshall (2009, henceforth SM), echoing earlier work by Treguier (1999), suggested that although $K_{eff}$ is small in the core of the ACC at the sea surface, it might be expected to be enhanced near the depth of the steering level of baroclinic waves growing on the thermal wind shear of the ACC. Using linear quasi-geostrophic stability analysis of a hydrographic climatology of the Southern Ocean, SM showed that the steering level of the fastest growing unstable modes resides at a depth of order 1.5 km and is roughly coincident with the level at which the meridional quasi-geostrophic potential vorticity (QGPV) gradient changes sign. Linear theory (Green 1970; Marshall 1981; Killworth 1997) suggests that the eddy diffusivity of a growing baroclinic wave has a maximum at the steering level and, in calculations with a non-linear stacked QG model, SM confirmed that this linear result
survives in the nonlinear regime. They also presented observational evidence that the phase speed of surface altimetric signals, the surface signature of interior baroclinic instability, propagate downstream at, roughly, 2 cm s\(^{-1}\), the speed of the mean current at a depth of 1.5 km or so, and much slower than the 10 cm s\(^{-1}\) mean surface current.

Here our goal is to map the meridional and depth structure of \(K_{\text{eff}}\) in the Southern Ocean, again using the effective-diffusivity methodology set out by Nakamura (1996). In the absence of observed three-dimensional velocity fields, we make use of an eddying numerical state-estimate of the Southern Ocean tightly constrained by observations, and we diagnose \(K_{\text{eff}}\) from the tracer distribution on isopycnal surfaces. The resulting effective-diffusivity cross sections support the notion of intensified mixing at depth and also reveals that deep mixing below the ACC connects with the heightened surface mixing found by MSJH on the equatorward flank. The structure of \(K_{\text{eff}}\) contains the signature of a critical layer, wherein the interplay between upstream-propagating waves and eastward mean flow determines where mixing is enhanced and suppressed.

Our paper is organized in the following way. Section 2 contains a description of the state estimate and the machinery used to calculate effective diffusivity. The results of the calculation and a discussion of the mixing patterns observed, along with some regional calculations, comprise Section 3. In Section 4, we present some new potential vorticity climatologies and show their relationship to the effective diffusivity. We also use \(K_{\text{eff}}\) in conjunction with the potential vorticity field to infer the eddy stress in the ACC region. A discussion of our findings and conclusions are given in Section 5.
2. Numerical Simulation of Tracer Transport

a. Southern Ocean State Estimate

This study takes advantage of a new generation of data products made possible by the ECCO project\(^1\). Oceanic state estimation, as reviewed by Wunsch and Heimbach (2006), provides a dynamically consistent framework in which to synthesize diverse observations, taking account of data uncertainties in a rigorous way. This is accomplished by iteratively optimizing a weighted least-squares fit between the observations and a general circulation model, in this case the MITgcm (Marshall et al. 1997).

We make use of the coarse-resolution (one-degree) Ocean Comprehensive Atlas (OCCA) of Forget (2008) which combines information in an ocean model with data from Argo subsurface floats, infrared and microwave sea surface temperature satellite measurements, along-track Sea Level Anomaly satellite altimetry measurements, and surface mean dynamic topography from the GRACE project. Monthly climatologies of temperature, salinity and associated currents are available for the years 2004-2006. The misfit between OCCA and the raw observations compares favorably to older global climatological data products such as NOAA’s World Ocean Atlas (Stephens et al. 2001) and Gourestski and Kolterman (2004). This data set was used to determine the mean hydrography of the Southern Ocean and to diagnose the mean potential vorticity.

To obtain eddying velocity fields with which to drive our advection-diffusion simulation, we employ the Southern Ocean State Estimate (acronym SOSE, Mazloff 2008), with a hori-

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\(^1\)Estimating the Circulation and Climate of the Ocean. Information available online at http://www.ecco-group.org .
Horizontal resolution of 1/6 degree. In addition to the above observations, SOSE is constrained by in-situ hydrographic data from CTD sections and SEaOS (southern elephant seals as oceanographic samplers) observations, all together over 1 billion optimization terms. A snapshot of the surface velocity field, revealing its rich mesoscale structure, is shown in Fig. 1 (b). We also calculated potential vorticity from the SOSE annual mean fields and compared them with OCCA; fronts are sharper in SOSE, but the overall structure and magnitude are quite similar. We have opted for using the OCCA dataset to describe the climatological state of the Southern Ocean, but our results would not look qualitatively different had we employed the mean SOSE solutions.

b. Tracer Advection

We characterize the eddy mixing by studying the evolution of a tracer governed by an advection-diffusion equation. The eddies stir the tracer, lengthening its contours and thereby enhancing the effect of small-scale diffusion. Nakamura (1996) developed a method to quantify this process by formulating the tracer equation in terms of a quasi-Lagrangian tracer-area coordinate, in which all transport is diffusive, making it possible to diagnose an effective eddy diffusivity using only a snapshot of the tracer field. Here we use the form given by Marshall et al. (2006), in which the effective diffusivity is written as:

$$K_{eff} = \kappa \frac{L_{eq}^2}{L_{min}^2}$$

(1)

where \(\kappa\) represents the small-scale diffusion that halts the cascade of tracer variance, \(L_{min}\) represents the length of an unstrained contour, and \(L_{eq}\), the equivalent length, can be thought of as the length of the strained contour. This equivalent length can be computed from an
instantaneous snapshot of the tracer field as

\[ L_{eq}^2 = \frac{\partial}{\partial A} \int |\nabla^2 q| dA \left( \frac{\partial q}{\partial A} \right)^2. \]  

(2)

We have included this expression for completeness, but we refer the reader to the Appendix of MSJH for its derivation.

The effective diffusivity formalism is rigorously defined for advection-diffusion of a tracer in two dimensions. However, we wish to obtain information about the vertical and meridional distribution of \( K_{eff} \). We therefore first employ the SOSE eddying velocity fields \( \mathbf{v} = (u, v, w) \) to advect a passive tracer \( q \) according to the 3D advection-diffusion equation

\[ \frac{\partial q}{\partial t} + \mathbf{v} \cdot \nabla q = \kappa_h \nabla^2_h q + k_v \frac{\partial^2 q}{\partial z^2} \]  

(3)

where \( \kappa_h \) (\( k_v \)) is a horizontal (vertical) diffusion coefficients and \( \nabla^2_h \) is the horizontal Laplacian. In a second step, the tracer field is then mapped onto two-dimensional neutral surfaces in the interior and \( K_{eff} \) evaluated from Eq.(1) using an appropriate choice of \( \kappa_h \), as described below.

Following MSJH, we chose an initial tracer distribution approximately aligned with the streamlines of the ACC as follows. Following Karsten and Marshall (2002), a single streamline of the vertically-integrated-transport streamfunction was chosen from the core of the ACC in the OCCA climatology. This was used as a reference to initialize tracer concentrations ranging from 0 to 1 along lines running parallel to this reference contour, as shown in Fig.1 (b). The same initial concentration was used on each vertical level. This choice leads to a rapid equilibration of the \( L_{eq} \) tracer contours, but any initial tracer gradient roughly perpendicular to the ACC would result in a reliable calculation. (This was confirmed by
repeating our calculations with the initial tracer contours simply aligned with latitude circles; the resulting $L_{eq}$ was not significantly different from the results presented here.) We also employ the contours of this initial tracer field to define an approximate “streamwise average.”

We performed the tracer advection on the same numerical grid as the original SOSE model, using the offline capability of the MITgcm. With grid points every 1/6th of a degree, the maximum grid spacing was approximately 18 km. Several experiments were carried out in which $\kappa_h$ was set to, respectively, 50, 100, 200, and 400 m$^2$ s$^{-1}$. In all cases the vertical diffusion is set to $\kappa_v = 1 \times 10^{-5}$ m$^2$ s$^{-1}$.

The role of diffusive processes inherent in the numerical implementation, together with the overall impact of horizontal versus vertical diffusion, can be assessed by considering the globally-averaged tracer variance equation:

$$\frac{1}{2} \frac{\partial \langle q^2 \rangle}{\partial t} = -\kappa_h^{(num)} \langle |\nabla_h q|^2 \rangle - \kappa_v^{(num)} \langle q_z^2 \rangle,$$

where $\langle \rangle$ indicates a volume-weighted average over the whole domain. Here $\kappa_h^{(num)}$ and $\kappa_v^{(num)}$ represent the diffusivities required to bring the observed decay in $q^2$ into consistency with the above variance equation. For all values of $\kappa_h$ in our experiments, the vertical term was at least an order of magnitude smaller than the horizontal, indicating that horizontal diffusion, rather than vertical, is responsible for dissipating tracer variance at small scales. More detailed analysis of the variance equation indicates that, for values of $\kappa_h$ below 200 m$^2$ s$^{-1}$, implicit numerical diffusion augments the explicit value of $\kappa_h$ by up to 60%, a finding consistent with incomplete resolution of the Batchelor scale. In particular, using $\kappa_h = 50$ m$^2$ s$^{-1}$ gave $\kappa_h^{(num)} = 83$ m$^2$ s$^{-1}$ while $\kappa_h = 100$ m$^2$ s$^{-1}$ gave $\kappa_h^{(num)} = 128$ m$^2$ s$^{-1}$. In these
cases, we use the estimated numerical value of $\kappa_h$ in calculating $K_{eff}$. Higher values of $\kappa_h$ did not generate significant spurious diffusion.

Before calculating $K_{eff}$, we allowed the tracer to evolve for one year. A sample tracer field after one year of stirring using $\kappa_h = 50 \text{ m}^2 \text{s}^{-1}$ is shown in Fig. 1 (b). Visual inspection reveals smooth, plausible tracer patterns and no evidence of grid-scale aliasing, despite the rather low value of diffusivity employed. Such levels of explicit mixing were also found to be appropriate in the study of MSJH, where altimetric fields were used to drive the tracer evolution rather than, as here, model fields constrained by observations.

c. *Isopycnal Projection*

The effect of mesoscale eddies acting in the “surface diabatic layer”, where isopycnals outcrop, differs fundamentally from their role in the adiabatic interior of the ocean (Held and Schneider 1999; Kuo et al. 2005). In the surface layer, eddies transport buoyancy horizontally across isopycnals. In the interior eddies stir primarily along tilted neutral surfaces, diminishing the role of eddy buoyancy fluxes. However, in the interior the eddies still play an important dynamical role, mixing potential vorticity and other tracers isopycnally. Here we attempt to characterize these two regions separately, diagnosing a horizontal $K_{eff}$ near the sea surface and an isopycnal $K_{eff}^{(i)}$ in the interior.

To calculate $K_{eff}^{(i)}$, we project the tracer values onto neutral surfaces. Neutral density $\gamma^n$ was calculated from the SOSE potential temperature and salinity fields using the algorithm of Jackett and McDougall (1997). We chose 35 values of $\gamma^n$ to provide sufficient vertical resolution. In each calculation, the tracer concentration after one year of forward integration
was interpolated to $\gamma^n$ surfaces. Note that Cerovecki et al. (2008) used a very similar approach to diagnose $K_{eff}^{(i)}$ in an idealized model of a baroclinically unstable zonal jet.

3. Cross-Sections of Effective Diffusivity in the Meridional Plane

Both the theoretical framework for deriving $K_{eff}$ in terms of the modified-Lagrangian-mean tracer equation and the numerical technique for its computation are well documented (Nakamura 1996; Nakamura and Ma 1997; Shuckburgh and Haynes 2003; Marshall et al. 2006) and so are not repeated here. We calculated $L_{eq}^2$ as defined in (2) using a MATLAB code. $L_{eq}^2$ was determined on both horizontal and isopycnal tracer surfaces from simulations using $\kappa_h$ values of 50, 100, 200, and 400 m$^2$ s$^{-1}$, yielding a total of eight cross-sections. To calculate $L_{min}^2$, the minimum possible length of a tracer contour was inferred by performing an experiment using $\kappa_h = 4 \times 10^4$ m$^2$ s$^{-1}$. This very large value of diffusivity decreases the Péclet number to the extent that explicit diffusion rather than advection dominates the tracer evolution. MSJH showed that in this regime, the resulting contour lengths, again calculated from (2), tend to $L_{min}$. $K_{eff}$ was then computed from (1). As described earlier, the level of mixing experienced by the numerical tracer, $\kappa_h^{(num)}$, was diagnosed from the tracer variance equation. A cutoff minimum was imposed on $L_{min}$ of 10,000 km, chosen to prevent small values of $L_{min}$ (caused by the surface outcropping of isopycnals or by the intersection of topography) from unrealistically inflating $K_{eff}$. Our results are not very sensitive to the method of computing $L_{min}$. The role of this factor is merely to provide a magnitude for
the effective diffusivity. In fact, we performed calculations simply using the length of a latitude circle for $L_{\text{min}}$ (accounting for the presence of land), and found more-or-less the same picture.

Both $L_{\text{eq}}$ and $L_{\text{min}}$ are defined as functions of the area $A$ enclosed by a tracer contour. A mapping exists between $A$ and an equivalent latitude $\phi_e$. In the atmosphere, in the absence of topography, this mapping simply identifies the latitude circle which encloses the given area. But here we must account for basin geometry as well as isopycnal outcrops. We can define the area enclosed by a latitude circle $\phi_e$ on a neutral surface $\gamma^n$ as

$$A_{\gamma^n}(\phi_e) = \int \int_{90^\circ S}^{\phi_e} g(\lambda, \phi) dA$$

where $g(\lambda, \phi) = 0$ for all locations not in the water, i.e. inside topography or beyond isopycnal outcrops, and $g = 1$ otherwise. We evaluated this expression numerically in the SOSE domain and used it to map $L_{\text{eq}}$ and $L_{\text{min}}$ to positions in latitude.

a. *Global Cross-Section*

The results of our calculations are shown in as meridional cross-sections in Fig. 2. (The isopycnal calculations were mapped back to depth coordinates using the average depth of the neutral surfaces at each latitude.) The horizontal and isopycnal diffusivities share certain characteristics. Each panel in Fig. 2 indicates a region of intense mixing deep beneath the ACC (centered around 54° S) which becomes shallower moving equatorward. The greatest differences between them occur, unsurprisingly, near the surface. The horizontal diffusivities tend to intensify near the surface, at least equatorward of the ACC region, while the isopycnal diffusivities uniformly decrease at depths shallower than 200 m because the sea surface acts
to suppress isopycnal stirring on tilted neutral surfaces.

From Fig. 2, we see that increasing the value of \( \kappa_h \) blurs the structure of \( K_{eff} \) somewhat. Indeed, visual inspection of the tracer fields reveals that (not surprisingly) they become increasingly smooth and less complex as \( \kappa_h \) is increased. Fig. 3 plots (horizontal) \( K_{eff} \) at the base of the mixed layer for various choices of \( \kappa_h \), along with the \( K_{eff} \) profile obtained by MSJH directly using altimetry. We clearly see that the distribution of \( K_{eff} \) obtained here using \( \kappa_h = 50 \) and \( 100 \) m\(^2\) s\(^{-1}\) are very close to those of MSJH, while those obtained using larger values if \( \kappa_h \) are considerably greater in magnitude. We therefore choose to analyze the case of \( \kappa_h = 100 \) m\(^2\) s\(^{-1}\), believing that it best represents the likely spatial distribution of \( K_{eff} \). This choice is also supported on consideration of the \((Pe, Nu)\) plot presented in MSJH, where \( Pe \) is the Péclet number and \( Nu = K_{eff}/\kappa_h \) is the Nusselt number. For sufficiently large \( Pe \), (i.e. sufficiently small \( \kappa_h \)) the slope of the line in \((Pe, Nu)\) space is order unity, in which case \( K_{eff} \) becomes independent of the small scale value of \( \kappa_h \) (MSJH, Shuckburgh and Haynes 2003). This increasing insensitivity can be clearly seen in Fig. 2 as \( \kappa_h \) is reduced.

Since we expect \( K_{eff} \), the horizontal diffusivity, to apply to eddy buoyancy fluxes in the surface diabatic layer, and \( K_{eff}^{(i)} \) to apply to isopycnal mixing in the interior, in Fig. 4 we present a composite of these two quantities. Ideally the boundary between the two regions would be drawn at the streamwise-averaged depth of the surface diabatic layer. Here we simply separate the regions at 100 m depth. Contours of the streamwise-averaged OCCA flow speed \( \sqrt{u^2 + v^2} \) are also shown in Fig. 4, and indicate the mean position of the ACC jet. A striking feature is that, in both the surface layer and the interior, effective diffusivity is significantly reduced where the mean flow is strongest. Mixing is enhanced between the
2 and 4 cm s$^{-1}$ contours, particularly on the equatorward side of the jet: the surface of maximum $K_{e,ff}$ is at a depth of order 1500 m beneath the core of the ACC and shallows on the equatorial flank.

SM anticipated this general form for $K_{e,ff}$, drawing on insights from linear baroclinic instability theory. In the analysis of a growing baroclinic disturbance (see, for example Green 1970; Marshall 1981; Killworth 1997), the diffusivity of quasigeostrophic potential vorticity can be shown to take the form

$$K_q = \frac{c_i}{k} \frac{\frac{1}{2}|\psi'|^2}{(U - c_r)^2}$$

(6)

where $U$ is the mean zonal current, $c_r$ is the real part of the phase speed, $c_i$ is the imaginary part (the growth rate), $k$ is the zonal wavenumber, and $\frac{1}{2}|\psi'|^2$ is the eddy kinetic energy. At finite amplitude, the general dependence of the diffusivity on the eddy kinetic energy, the phase speed, and the mean flow can be expected to hold. SM noted that the observed zonal propagation speed of altimetric signals in the ACC was roughly 2 cm s$^{-1}$, significantly smaller than mean surface zonal currents, typically 10 cm s$^{-1}$ (shown, for example, in Fig. 4). At depth, however, where mean flow advection and wave propagation speeds are much more closely in sink, the wave can “keep up” with the mean flow and achieve large meridional excursions of fluid parcels, promoting mixing. SM followed up these linear arguments with detailed, fully turbulent calculations with a stacked quasi-geostrophic model, which was relaxed back to observed hydrography on the large scale. They confirmed that intensified mixing of potential vorticity occurred at depth, near the critical level predicted by linear theory where $U = c$. The findings reported here, which make use a much more realistic eddying model constrained to be close to observations, support the idea that critical-layer
enhancement occurs in the region of the ACC. The numerical studies of Treguier (1999) and Cerovecki et al. (2008) also provide clear evidence of intensified mixing in the critical layer of a baroclinically unstable jet. Treguier (1999) in particular diagnoses mixing coefficients based on flux-gradient inversions of both quasi-geostrophic potential vorticity in the horizontal and Ertel potential vorticity along isopycnals. The resulting vertical diffusivity profile (her Fig. 9) is remarkably similar to our vertical profile of $K_{eff}^{(i)}$ in the jet axis, reaching a peak of 1600 m$^2$ s$^{-1}$ at 1500 m depth.

If critical-layer effects are responsible for enhanced diffusivity peak at depth, we might expect to observe eddies propagating eastward at a speed slower than the surface mean flow in SOSE. The phase speeds can be calculated using Radon transforms, as done in SM, but here we opt for simpler approach of constructing Hovmueller diagrams. We examined the SOSE sea-surface height anomaly in a sector in the Pacific between 165° W and 135° W. Fig. 5 shows two Hovmueller diagrams, one at 53° S, near the mean zonal flow maximum in this region, and on at 47° S. It is encouraging to see that $c \simeq 2$ cm s$^{-1}$ in the ACC, since this places the steering level around 2000 m deep in agreement with the structure of $K_{eff}$ in Fig. 4. North of the jet at 47° S, the anomalies propagate westward at approximately 1 cm s$^{-1}$. These numbers are in agreement with those of SM.

b. Regional Cross-Sections

While the global cross-section of effective diffusivity seems to offer a picture consistent with the global streamwise average mean fields, the Southern Ocean contains large zonal asymmetries in bathymetry, circulation, and eddy activity. To address this the zonal vari-
ations in mixing, we split the domain into six sectors and repeated the calculation on each sector. Shuckburgh et al. (2008b) showed how this procedure, while not formally permitted in Nakamura’s construction, still gives meaningful values of $K_{eff}$ in the truncated domain.

The cross sections of the isopycnal diffusivity are shown in Fig. 6, along with zonally-averaged zonal velocity and isopycnals. (Streamwise averages are unnecessary in the smaller domain.) Intensified mixing at depth is clearly present on the flanks of the jet maxima in every sector, the location in latitude varying with the local flow. Other regions of enhanced mixing in each sector can also be related to the local current system. In particular, as noted by Shuckburgh et al. (2008b), strong mixing is found in regions where eddies are generated in association with topographic features in regions of weak zonal mean flow. In Fig. 6(a), the region south of Africa between the Atlantic and Pacific, the mixing in the ACC is concentrated in a narrow region below and equatorward of the jet. Another surface-intensified mixing region appears north of 40° S. Movies of the tracer evolution suggest that this mixing is associated with the intense eddies of the Agulhas rings. The Indian Ocean sector, Fig. 6(b), shows very strong mixing on both sides of the jet as well as below. This sector contains the Kerguelen Plateau, a large topographic feature that generates strong eddy activity as the flow passes over and around it. South of Australia, Fig. 6(c), the only strong mixing occurs in a deep, narrow band between 1000 and 2000 m depth. Fig. 6(d) is the south-west Pacific, the region in which the Hovmueller diagrams of Fig. 5 were constructed. In agreement with our critical layer hypothesis, enhanced mixing is observed at around 53° S below the jet where the zonal velocity is $2 \text{ cm s}^{-1}$, and at 47° S where the zonal velocity is $1 \text{ cm s}^{-1}$. In Fig. 6(e), the southeast Pacific, $K_{eff}$ seems quite weak, consistent with the low eddy kinetic energy in this region and with the results of Shuckburgh et al. (2008b) however,
it still shows intensification with depth. Finally, downstream of Drake passage (Fig. 6f),
intense mixing appears very widespread. It is likely that much of the mixing north of 45° S
is the result of eddies spawned by the Falkland current.

The Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean (DIMES) will
take place from 2009 to 2012 (Gille et al. 2007). The goal of the experiment is to measure
mixing rates in the region upstream of the Drake Passage using RAFOS floats and a patch
of passive tracer. Our results predict that isopycnal mixing rates vary considerably with
latitude and depth. In order to gain some insight into the locations of intensified mixing
in the DIMES region, we can map the effective diffusivity back onto tracer contours from
the tracer snapshot used in the $K_{eff}$ calculation, since $K_{eff} = K_{eff}(A) = K_{eff}(q)$. We have
constructed such maps on the two DIMES isopycnals, $\gamma^{n} = 27.2$ and $\gamma^{n} = 28.9$, in Fig. 7.
(These isopycnals are also highlighted in Fig. 6f.) Satellite altimetric data will be available
the DIMES investigators in real time during the initial deployment of floats and tracer. With
this in mind, we have included contours of the instantaneous sea-surface height from SOSE
in Fig. 7, which suggest the position of the ACC fronts. This presentation highlights the
fact that isopycnal mixing on the deeper surface is strongest directly below the front, while
on the shallower surface the mixing is weak below the front and stronger to the north.
4. Observed Relationship Between $K_{eff}$ and Potential Vorticity Distributions

Potential vorticity (PV) behaves like a conserved tracer in the ocean interior. The same mixing processes acting on the passive tracer in our experiment act on the PV field. Rhines and Young (1982) showed how eddies tend to homogenize subsurface tracers such as PV through mixing. Thus we might expect PV homogenization in regions where $K_{eff}$ is large. Furthermore, by inferring the eddy flux of PV based on effective diffusivities, we can further explore the dynamical implications of critical-layer mixing enhancement.

a. The Potential Vorticity Field

We diagnosed the potential vorticity distribution using the OCCA climatology. The following approximate form is computed:

$$P = f \frac{\partial b}{\partial z}$$  \hspace{1cm} (7)

where $b$ is the buoyancy and $f$ the Coriolis parameter. The relative vorticity has been neglected because the Rossby number is small on the large scales. Using the same isopycnal transformation described in Section 2, we computed $P$ and took its streamwise average in buoyancy space, i.e. following streamlines and isopycnals. The result is plotted, transformed back to depth coordinates, in Fig. 8. We see that surfaces of constant $b$ and surfaces of constant $P$ align with one another over much of the domain. This is confirmed by the plot of the isopycnal gradient of $P$ shown in Fig. 9a, which is small in much of the interior. Notably, PV gradients become very large near the surface in the southern ACC region, just where we
observe reduced effective diffusivities. To examine this more closely, in Fig. 10 we plot the pv gradient along with $K^{(i)}_{\text{eff}}$ on the neutral surfaces 28.1, 28.0, 27.9, 27.8, and 27.6. We do indeed find consistently low values of $K^{(i)}_{\text{eff}}$ where PV gradients are low and vice-versa. Where the PV gradient is very weak, no amount of mixing can lead to a PV flux. However, there also appears to be a transition zone, where both $\partial P/\partial y_b$ and $K_{\text{eff}}$ are nonzero. This suggests a nonzero eddy flux of PV. In what follows, we investigate whether the PV fluxes implied by these gradients and $K_{\text{eff}}$ are consistent with common ideas of eddy-mean-flow interaction in the Southern Ocean.

b. The Eddy Stress

We consider the streamwise- and time-average momentum balance of the large-scale flow through the Transformed Eulerian Mean (TEM) zonal momentum equation (see, e.g. Andrews et al. (1987))

$$-fv_{\text{res}} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \bar{v}' q' + \frac{1}{\rho_0} \frac{\partial \tau^w_x}{\partial x},$$

(8)

where $v_{\text{res}}$ is the residual meridional velocity, a sum of eddy and mean contributions, $\bar{v}' q'$ is the meridional eddy quasi-geostrophic potential vorticity (QGPV) flux, and $\tau^w_x$ is the wind stress.

Following, for example, Green (1970); Rhines and Young (1982); Killworth (1997); Visbeck et al. (1997); Treguier et al. (1997), etc., we can try to represent $\bar{v}' q'$ as a transfer down the mean gradient $\overline{q_y}$. Rather than trying to infer the diffusivity from scaling arguments or turbulence theory, we will use our isopycnal effective diffusivity, resulting in the
Why not use the horizontal effective diffusivity, since the equations are in height coordinates, rather than the isopycnal effective diffusivity? As shown by Plumb and Ferarri (2005), in the limit that relative vorticity can be neglected, the residual meridional isentropic eddy flux of Ertel PV is proportional to the horizontal meridional eddy flux of QGPV. Indeed we note in passing that, under quasigeostrophic scaling, the meridional gradient of QGPV at constant $z$ is equal to the meridional gradient of Ertel potential vorticity $P$ along isopycnals: i.e.

$$\left( \frac{\partial q}{\partial y} \right)_z \simeq \frac{1}{N^2} \left( \frac{\partial P}{\partial y} \right)_b.$$  

We checked this relation in OCCA and found it to hold well everywhere outside of the mixed layer. Thus Eqs. (8&9), written down in height coordinates, are not as restrictive as they may seem: they are isomorphic, both mathematically and physically, to analogous expressions in isopycnal coordinates. Moreover, since our isopycnal effective diffusivities most closely describe the transport of a conserved tracer (like Ertel PV) along isopycnals, $K^{(i)}_{eff}$ is the more appropriate choice to capture the horizontal eddy mixing of QGPV. We do note, however, that the use of $K_{eff}$ at all is not formally justified in the TEM framework or indeed in any Eulerian context.

We computed $q_y$ at constant $z$ using the OCCA dataset thus (where relative vorticity has been neglected)

$$\frac{\partial q}{\partial y} = \frac{\partial}{\partial y} \left[ f + \frac{\partial}{\partial z} \left( \frac{f}{N^2} b \right) \right] = \beta - f \frac{\partial s}{\partial z},$$  

and $s = -b_y/N^2$ is the isopycnal slope. Numerical implementation requires much differentiation and can lead to noise on the gridscale which was removed using a Shapiro filter. The
streamwise-averaged $\bar{q}_y$ is shown in Fig. 9b.

As discussed, for example, in Ferreira et al. (2005) it is useful to write the eddy forcing term in Eq.(8) as the vertical divergence of an eddy stress $\tau^e_x$ (sometimes called form drag) thus:

$$\bar{v}'q' = \frac{1}{\rho_0} \frac{\partial \tau^e_x}{\partial z}$$ (12)

We compute $\tau^e_x$ from (12) by integrating $\bar{v}'q'$ (obtained using $K_{\text{eff}}$ from Eq. 9) down from the surface to depth. The eddy stress is thus given by

$$\tau^e_x = -\rho_0 \int_0^z \bar{v}'q'dz = \rho_0 \int_0^z K^{(i)}_{\text{eff}} \frac{\partial \bar{q}}{\partial y} dz .$$ (13)

The resulting distribution of $\tau^e_x$, along with the wind stress $\tau^w_x$, is shown in Fig. 11a. The result of the parameterization is a stress pattern that increases monotonically from the surface and is greatest on the poleward edge of the ACC, slightly south of the latitude where the wind stress peaks. The maximum value of the eddy stress, around 0.2 N m$^2$, is close in magnitude to the maximum wind stress. The deduced eddy stress supports the notion that eddies act to transfer momentum from the wind down the water column, where it can finally be balanced by topographic form stress (Johnson and Bryden 1989; Olbers et al. 2004; Ferreira et al. 2005). It is encouraging that our parameterization using $K_{\text{eff}}$ was skillful enough to achieve an appropriate magnitude for the eddy stress without resorting to any arbitrary “tuning” constants. A likely form of $\tau^e_x$ was inferred by Ferreira et al. (2005) by using it as a control parameter in a minimization procedure to reduce the misfit between a global ocean model and climatological observations. Their inferred stress in the Southern Ocean was similar in magnitude to ours, but their $\tau^e_x$ reached its vertical maximum near 200 m depth. Our maximum stress is found closer to 2000 m depth.
In Fig. 11b, the eddy stress implied by a constant QGPV diffusivity $K = 1000 \text{ m}^2 \text{s}^{-1}$ is shown for comparison. The constant QGPV diffusivity parameterization is equivalent to the Gent and McWilliams (1990) parameterization under certain limits (e.g. Wardle and Marshall 2000). The most obvious result of this comparison is that the two parameterizations yield quite similar results. One clear difference is that the constant diffusivity parameterization produces high values of eddy stress farther south. It also causes the eddy stress to reach high values at slightly shallower depths. Overall, these differences are more due to the low values of $K_{eff}$ in the ACC core and on the poleward flank than to the high values in the critical-layer regions. This results from the fact, noted earlier, that the regions of highest mixing correspond with very weak PV gradients.

5. Discussion and Conclusions

This study has used a tracer based approach, together with a model of the Southern Ocean constrained by observations, to quantify mesoscale eddy stirring in the Southern Ocean. The foremost result we wish to highlight is the presence of enhanced eddy effective diffusivity well below 1000 m depth in the Southern Ocean. Overall, a structure is observed of reduced eddy diffusivity at the core of the ACC, with enhanced values on its flanks where the zonal velocity is in the range 1-3 cm s$^{-1}$. We suggest that this is strongly indicative of mixing at a critical layer. These findings are consistent with the quasi-geostrophic analysis of SM and the idealized models of Cerovecki et al. (2008) and Treguier (1999), and with theoretical considerations based on the observed reversal of potential vorticity gradients and the steering level of linear baroclinic modes (SM).
Shuckburgh et al. (2008a) and Sallée et al. (2008) noted that at the surface, assessments of eddy mixing based on particle dispersion do not indicate a minimum at the core of the ACC. Finite-time Lyapunov exponents, another possible measurement of eddy stirring, correlated closely with eddy kinetic energy but do not seem to be diminished by strong mean flow, reaching their highest values in the western boundary currents and ACC (Waugh 2008; Shuckburgh et al. 2008a,b). Similarly, particle based results do not seem to indicate enhanced mixing below the ACC at depth (Griesel et al. 2008). In light of the upcoming DIMES experiments, it is important to reconcile these views of eddy mixing. Some progress has been made on this front by d’Ovidio et al. (2009) and Shuckburgh et al. (2009).

We argue that effective diffusivity is a useful metric because of the dynamically consistent relationship it holds with the IPV gradient. The strong IPV gradient in the core of the ACC acts as a barrier to wave propagation—baroclinically unstable waves propagating in the steering level on the edge of this gradient break, form closed eddies, and homogenize the low-PV region equatorward of the jet. This conceptual picture bears a close resemblance to the winter stratosphere, where high effective diffusivities in the “surf zone” outside the polar vortex are indicative of breaking planetary waves (Haynes and Shuckburgh 2000). However, several important distinctions complicate this interpretation. The size of mesoscale eddies in the Southern ocean is far below the planetary scale. Indeed, the streamwise-averaged view necessarily obscures localized eddy processes such as interactions with particular topographical features (e.g. the Aghulas rings). This point is underscored by the wide variations seen in the effective-diffusivity patterns between sectors. Secondly, planetary waves in the stratosphere are generally thought to propagate up from the troposphere (McIntyre and Palmer 1983), far below the critical layer they encounter in the stratosphere. There is no
such dynamical separation in the ACC. The eddies arise as a result of baroclinic instability and interact with the critical layer in the same region, extracting momentum from the mean state and then depositing it again when they break. Perhaps a better atmospheric analogy is the midlatitude troposphere, where intense mixing at the steering level homogenizes PV and brings the mean state closer to neutrality. This processes has been studied in the context of baroclinic adjustment (Stone 1978; Zurita-Gotor and Lindzen 2004a,b).

We have used our calculated effective diffusivity to infer the eddy stress. This produced a reasonable picture; the eddy stress had the appropriate magnitude and position to balance the wind stress. The parameterization using effective diffusivity was compared to a constant diffusivity parameterization or roughly similar magnitude (1000 m$^2$ s$^{-1}$). We found the greatest differences to arise from the low values of $K_{eff}$ in the ACC core and on its poleward flank, rather than from the high values of $K_{eff}$ found at critical layers, where the weak PV gradients dominate. It appears that the critical layer mixing has already played its role by homogenizing the PV, helping to determine the equilibriated state represented by the OCCA climatology. The question of how the equilibrium was reached is inaccessible to the methods of this study, but perhaps studying the spin-up of a zonal jet could provide some answers. More generally, the consequences of the variations of the effective diffusivity for the large-scale circulation merit further study.

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