IAP - 12.310

Introduction to Weather Forecasting

Lecture 2: Relationship between the mass and wind

- upper air map
Today: 850 mb Temperature °C
Pole

Eq

Warm

Cold

z

p1(850)

p4(300)

p3(500)

p2(700)

p1(850)
Upper level Flow
A case study: 080109 06z
080109 06z - 850 mb Temperature °C
Does the earth rotation matter?

• Rossby number: \[ R_o = \frac{\tau_{\text{earth}}}{\tau_{\text{phenomenon}}} \]

• Phenomenon: the jet stream

• How long does it take for an air particle in the jet to go around the full globe?
Earth radius = 6378 km

Circle around the globe at 45 lat = \( 2\pi R \cos \vartheta \)

\(~ 28,000 \text{ km} \)

Typical velocity in the jet \( u = 30 \text{ m/s} \)

Typical timescale

\[ \tau_{\text{phenomenon}} = \frac{28,000 \text{ km}}{30 \text{ m/s}} \sim 9,000 \text{ s or 10 days} \]

\[ \tau_{\text{phenomenon}} \]

\[ \tau_{\text{earth}} \]

\[ R_o = \frac{\tau_{\text{earth}}}{\tau_{\text{phenomenon}}} \sim 0.1 \]

small Rossby number implies earth rotation important
Earth rotation and the Coriolis Force

Example: a ring of air moving from west to east
Let's consider a ring of air moving from west to east with velocity $U$.

The Coriolis acceleration $C$ is pointing south.

The centrifugal acceleration $A$ is given by:

$$A = \frac{V^2}{r} = \frac{(U + \Omega r)^2}{r} = \Omega^2 r + 2\Omega U + \frac{U^2}{r}$$

To a good approximation:

$$A = 2\Omega U$$

'A' can be resolved into two components: B and C.

B' is perpendicular to the earth surface.
'B' changes the weight of the ring slightly.

'C' is parallel to the earth surface: $C = 2\Omega \sin \phi U$.

$C = 2\Omega \sin \phi U$ is the Coriolis acceleration and it's pointing south.

What other force is acting on the ring of air and pointing north?
Let’s postulate a balance between Coriolis force and the pressure gradient force:

\[ 2\Omega \sin \varphi U \times \varrho d\phi dz = -\frac{\partial p}{\partial \phi} d\phi dz \]

acceleration mass

\[ dy = a d\phi \]

\[ 2\Omega \sin \varphi U \times \varrho dydz = -\frac{\partial p}{\partial y} dydz \]

\[ 2\Omega \sin \varphi U = -\frac{1}{\varrho} \frac{\partial p}{\partial y} \]

\[ f = 2\Omega \sin \varphi \]
is the Coriolis parameter

\[ U = -\frac{1}{\varrho_f} \frac{\partial p}{\partial y} \]

\[ V = \frac{1}{\varrho_f} \frac{\partial p}{\partial x} \]

This is the geostrophic wind resulting from the balance between the PGF and the Coriolis force
Let’s postulate a balance between the pressure gradient force and the Coriolis force (aka Geostrophic Balance)

Mathematically, the geostrophic equations:

\[ f_u = -\frac{1}{\rho} \frac{\partial p}{\partial y} \]
\[ f_v = \frac{1}{\rho} \frac{\partial p}{\partial x} \]

\[ f = 2\Omega \sin \phi \]

This is the Geostrophic Wind
The geostrophic relationship is

in height coordinates:

\[ u = - \frac{1}{\rho f} \frac{\partial p}{\partial y} \]
\[ v = + \frac{1}{\rho f} \frac{\partial p}{\partial x} \]

in pressure coordinates:

\[ u = - \frac{g}{f} \frac{\partial h}{\partial y} \]
\[ v = + \frac{g}{f} \frac{\partial h}{\partial x} \]
Review geostrophic wind calculations at:

- upper level
- surface