In this assignment we will use upper air and surface analyses to study the connection between the wind and the pressure field on the large-scale to check how close they are to geostrophic balance.

Use GEMPAK to plot 500mb height and wind from recent analyses. Notice that the analyzed wind tends to follow the 500mb height with a speed inversely proportional to the separation of the height isolines as we expect from the geostrophic approximation.

The geostrophic relationship is:

in height coordinates:
\[ \mathbf{v}_g = \frac{1}{\partial f} \mathbf{k} \times \nabla p_{z=\text{const}} \]

in pressure coordinates:
\[ \mathbf{v}_g = \frac{g}{f} \mathbf{k} \times \nabla z_{p=\text{const}} \]

Use GEMPAK to plot the geostrophic wind and check how close it is to the observed wind.

Find examples of strong confluence and divergence in regions of jet streaks. How close is the wind to geostrophic balance at the jet entrance and exit regions?

Look at regions of large curvature, such as pronounced troughs or ridges. How close is the geostrophic wind to the observed one in these regions?
1 Accelerations tangential to the flow

Air may be accelerated in a direction tangential to the motion as, for example (Fig.1), along the central streamline of a straight jet.

By making use of the x-momentum equation along the direction of the jet:

\[
\frac{Du}{Dt} - f v_{ag} = 0
\]

deduce the sign of the ageostrophic wind at the entrance and exit to the jet.

Is your data consistent, both qualitatively and quantitatively?

2 Radial acceleration: Gradient wind

Here we will examine the effect of curvature in more detail by looking at flow in very strong cyclones - e.g. a hurricane.

In the second week of September'04 hurricane “Ivan”, one of the strongest hurricane of the season (cat 5), developed in the western Atlantic and moved towards the east and the Gulf of Mexico – see attached surface maps and satellite images for the 13\textsuperscript{th} and 14\textsuperscript{th} of September ’04.

Plot the geostrophic wind at various pressure levels in the cyclone and compare it to the analyzed wind. What do you find?

Let’s consider parcels of air that are trapped in closed, axi-symmetric motion, moving with constant tangential velocity $v_\theta$ along a path of curvature $r$ - see Fig.1.

If the Rossby number,

\[
R_o = \frac{|v_\theta|}{fr}
\]  

is of order unity, then the balance of forces in the radial direction is a three-way balance between centrifugal, Coriolis and pressure gradient forces:

\[
\frac{v_\theta^2}{r} + fv_\theta = g \frac{\partial h}{\partial r}
\]
Figure 1: The velocity of a fluid parcel viewed in the rotating frame of reference: 
\[ v_{rot} = (v_\theta, v_r) \].

where \( h \) is the height of a pressure surface (see Eq. (??) in Appendix). This can be re-arranged to give:

\[
v_\theta = \frac{g}{\left(f + \frac{v_\theta}{r}\right)} \frac{\partial h}{\partial r}
\]

Thus we expect that the observed wind will be less than the geostrophic wind in a cyclonic situation \((v_\theta > 0)\). Is your data consistent?

What about anticyclonic situations \((v_\theta < 0)\)?

Solving the above equation for \( v_\theta \) we find that:

\[
v_\theta = -\frac{1}{2} fr \pm \left(\frac{1}{4} f^2 r^2 + gr \frac{\partial h}{\partial r}\right)^{\frac{1}{2}}
\]

where the positive root must be chosen; why? Is there any quantitative agreement with your observed cyclone?

Compute the ratio of geostrophic to observed wind; how does this ratio depend on the Rossby number, Eq.(1), of your system?

What would you expect in a tornado?

2.1 Ekman Layer

How close to geostrophic is the wind at the surface? - plot mean sea-level pressure, deduce the geostrophic wind and compare it with the observed wind. Think about the balance of forces in the surface boundary layer
The surface wind stress produces important frictional forces in the surface boundary layer of the atmosphere, extending up to an elevation of about 1 km, which retard the boundary layer winds and induces cross isobaric flow.

The horizontal momentum equation in the surface boundary layer can be written as (see 12.003 notes):

\[ \frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + \frac{1}{\rho} \nabla p = \mathbf{F} \]

where the frictional force is \( \mathbf{F} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \) and \( \tau \) is the stress.

Supposing that \( \mathbf{v}_0 \) is the wind at the surface and that the frictional force over the boundary layer acts against it, sketch the balance of forces - frictional, pressure gradient and Coriolis - in the boundary layer. Mark on your diagram \( \mathbf{v}_0 \) and \( \mathbf{v}_g \), where \( \mathbf{v}_g \) is the geostrophic wind in balance with pressure gradient forces. Deduce that \( \mathbf{v}_0 \) must have a component directed down the pressure gradient.

If \( \tau_0 \), the frictional stress at the ground, is given by

\[ \tau_0 = -\rho c_d v_0 \mathbf{v}_0 \]

where \( c_d \) is the drag coefficient, show that \( \gamma \), the angle between \( \mathbf{v}_0 \) and \( \mathbf{v}_g \), is given by

\[ \gamma = \tan^{-1}\left( \frac{c_d v_0}{\Delta z f} \right) \]

and

\[ \frac{v_0}{v_g} = \sqrt{\frac{1}{1 + \left( \frac{c_d v_0}{\Delta z f} \right)^2}} \]

where \( \Delta z \) is the depth of the boundary layer and \( f \) is the Coriolis parameter.

If \( c_d \) has a value of, on the average, \( 2 \times 10^{-3} \), \( \Delta z = 10^3 m \) and \( f = 10^{-4} s^{-1} \), calculate \( \gamma \) and \( \frac{v_0}{v_g} \). Hence complete and discuss the table below (\( c_d \) varies from \( 2 \times 10^{-3} \) to about half this value over the oceans up to a few times this value over rugged mountains).
What happens in the limit that the frictional force overwhelms the Coriolis force?

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