12.804
Project 2
Quasi-geostrophic Potential Vorticity-
atmospheric data

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1 Quasi-geostrophic scaling

In this part of the assignment we estimate scales associated with a ‘synoptic’
system that is presently affecting US region (see attached surface and upper
air maps) and compare them to the quasi-geostrophic scaling.

(a) Produce zonal and meridional cross-sections of wind and potential tem-
perature across the synoptic system and estimate:

1. a tropospheric scale height, $H = \frac{RT_o}{g}$, where $T_o$ represents a mean
tropospheric temperature in the region of the system.

2. the buoyancy frequency, $N^2 = \frac{\mathcal{g}}{\vartheta_o} \left( \frac{\partial \vartheta}{\partial z} \right)$ for the troposphere and the layer
 bounded by 500 mb and 300 mb, where $\vartheta_o$ is a reference mean potential
temperature.

3. the Rossby radius of deformation, $L_D = \frac{NH}{\mathcal{f}}$.

4. the Richardson number, $Ri = \frac{N^2}{\left( \frac{\partial U}{\partial z} \right)^2}$ for the troposphere and for the
 layer bounded by 500 and 300 mb.

(b) Plot surface and upper air maps and estimate the zonal $L_x$ and meridi-
ional scales $L_y$ of the synoptic disturbance over the US area at the
surface and at 500 mb. Compare your estimates with $L_D$. Determine a mean zonal wind $U$ and estimate the Rossby number $R_o = \frac{U}{fL}$ and an advective timescale $\frac{L}{U}$ for the region bounded by latitudes $30^\circ N$ and $40^\circ N$ over the United States. Does $R_o R_i^{1/2} \approx 1$?

(c) You might find it interesting to check your scaling estimates using the facility in GEMPAK to compute, for example, the $R_o$ number, $N^2$, and $R_i$.

Useful information
$R = 287 \, J \, kg^{-1} K^{-1}$
$C_p = 1005 \, J \, kg^{-1} K^{-1}$

2 Computation of Quasi-geostrophic Potential Vorticity

We now go on to evaluate terms in the quasi-geostrophic potential vorticity equation. It can be shown that quasi-geostrophic potential vorticity $q_p$ is conserved following an isobaric trajectory for adiabatic and frictionless motion

$$D_g q_p = 0$$

(1)
where $D_g$ is the geostrophic operator $D_g = \frac{\partial}{\partial t} + V_g \cdot \nabla$ and $q_p$ is the quasi-geostrophic potential vorticity defined by:

$$q_p = (\zeta_g + f) + f_0 \frac{\partial}{\partial p} \left( \frac{\theta'}{S(p)} \right)$$

(2)
where $S$ is the static stability of a reference potential temperature profile
$S = \left( \frac{d\theta_0}{dp} \right)$
$\theta'$ is the deviation of the potential temperature from the reference $\theta_0(p)$
$\zeta_g + f$ is the absolute vorticity with
$f$ the planetary vorticity and
$\zeta_g$ the geostrophic relative vorticity evaluated on an isobaric surface.

(a) First evaluate the absolute vorticity at 400 mb using GEMPAK: $gfunc = avor(geo)$. 


(b) Evaluate the stretching term by finite difference over the 500 - 300 mb layer. The static stability $S$ is, in quasi-geostrophic theory, only a function of $p$; it must therefore be computed from vertical profiles of potential temperature horizontally averaged over the domain (see attachments which outline how GEMPAK can be used to compute the various terms).

(c) Add (a) and (b) to compute $q_p$.

Are the stretching and absolute vorticity terms of comparable magnitude? How large are the meridional gradients of $q_p$ relative to the planetary vorticity gradient?

Do you notice any closed $q_p$ contours? If so what is their physical significance?

Is there a tendency for the pressure contours and $q_p$ contours to follow one another?

Is the anomaly in $q_p$ consistent with the presence of an intense upper level trough? - think about the ‘invertibility principle’.

(d) Repeat a), b) and c) but at the later time. Use GEMPAK to evaluate the terms on the left hand side of the in the quasi-geostrophic potential vorticity equation:

$$\frac{\partial}{\partial t} q_p + V_g \cdot \nabla q_p = Sources + Sinks \tag{3}$$

Do they balance one another?

3 Appendix

Use an appropriate finite-difference expression to estimate the stretching term $f_0 \frac{\partial}{\partial p} \left( \frac{\partial^2}{S(p)} \right)$ at 400 m

1. Evaluate static stability $S = \left( \frac{d\theta}{dp} \right)$ between standard pressure levels, computing the area-averaged potential temperature using GEMPAK grid-area function $savs : gfunc = savs(thta)$
2. Evaluate $\frac{\nabla f}{\nabla \theta}$ at 300 and 500 mb using GEMPAK program GDDIAG which computes a scalar diagnostic grid and stores it in a grid file (give a name e.g. $tht$ to the new scalar diagnostic field).

3. Evaluate the stretching term at 400mb using $tht$ at 300 and 500 mb:
   
   \[ \text{stretch} = tht(300) - tht(500)/Dp \]

4. Evaluate the quasi-geostrophic pv at 400mb as: $q = avor + stretch$