Part II

The Ocean
Chapter 8

The ocean and its circulation

[Hartmann, Ch 7 (7.1-7.3).]

The ocean is, like the atmosphere, a fluid on a rotating Earth and the two have many similarities in their behavior and, especially, in their fluid dynamics. However, there are some important differences:

- The fluids are physically different: water is (almost) incompressible, and ocean thermodynamics has no counterpart of atmospheric moisture (as a source of latent heat).

- With the exception of a narrow gap in the Southern Ocean between the tip of South America and the northern tip of the Antarctic peninsula, all oceans are laterally bounded by continents - see fig(8.1). Thus, unlike the atmosphere, zonal currents are blocked by coasts (except in the Southern Ocean where the Antarctic Circumpolar Current extends all the way around the globe). It should be noted, however, that the ocean basins have evolved on geological timescales - and the distribution of land and sea in the past has been very different from today, with profound implications for ocean circulation and climate - see below.

- The ocean circulation is forced in a different way from the atmosphere. We have seen that the atmosphere is largely transparent to solar radiation and warmed from the ground. By contrast, buoyancy forcing of the ocean is from above.\(^1\) Although the ocean is rather opaque to visi-

\(^1\)There are sources of geothermal heating at the bottom of the ocean but, in an average sense, this accounts for only a few milliwatts/m² of heat input, compared to air-sea heat fluxes of ±10 to 100W/m² at the surface.
Figure 8.1: Bathymetric relief of the ocean basins. Depths in many places are greater than 6km.

...ble radiation (and very opaque to IR) it exchanges heat and moisture with the atmosphere at its upper surface (through transfer of sensible heat, latent heat through evaporation, and moisture through evaporation and precipitation) driving what is known as the ‘thermohaline’ circulation. In addition, there is a very important process forcing the ocean circulation that has no counterpart for the atmosphere. Winds blowing across the ocean surface exert a stress on the ocean, thus forcing the ‘wind-driven’ circulation of the ocean which, especially at upper levels, is a major component of the net circulation.

8.1 Physical characteristics of the ocean

The ocean covers about 71% of the Earth’s surface; average total depth is 3.7km - see the bathymetry plotted in fig.(8.1). This adds up to a huge amount of water (about $1.3 \times 10^{21}$kg) and a correspondingly enormous heat capacity, which is one of the reasons that understanding the ocean is so important to understanding climate.
Figure 8.2: The world pattern of plates, ocean ridges and trenches. Earthquake epicenters congregate along the boundaries of the plates, which are moving relative to one another at a rate of $\sim 5$ cm yr$^{-1}$.

The distribution of land and sea today, at the present stage of the evolution of the earth, can be seen in fig.(8.2). But the land distribution has been very different in the past, as can be seen from fig.(8.3) showing paleogeographic reconstructions from the Jurassic (170Ma ago), the Cretaceous (100Ma ago) and the Eocene (50ma ago). The circulation of the ocean is profoundly affected by the geometry of the land-sea distribution and so we can be sure that the pattern of ocean circulation in the past, and perhaps its role in climate, must have been very different from that of today.

The bathymetry of the present ocean basins - see fig.(8.1) - is highly complex and much more jagged than that of the land surface. We shall see that abyssal ocean currents are very weak and erosion of submarine relief consequently very slow.

### 8.1.1 Properties of seawater; equation of state

Unlike air, the density of sea water varies rather little - by only $\sim 3\%$ - but these variations turn out to be very important! In particular, sea water is almost incompressible; not quite, since at the enormous pressures in the deep
Figure 8.3: Panthalassa was the huge ocean that dominated one hemisphere. Pangea was the supercontinent in the other hemisphere. (a) Jurassic (170 Ma ago), (b) the Cretaceous (100 Ma ago) and (c) Eocene (50 Ma ago).
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Figure 8.4: Contours of seawater density anomalies ($\sigma = \rho - \rho_o$ in kg/m$^3$) plotted against salinity (g kg$^{-1}$) and temperature ($^\circ$C). Note that sea water in the open ocean has a salinity in the range 33-36 o/oo; see fig.8.7.

Ocean compressibility effects are not negligible. However, for our purposes we will henceforth neglect compressibility and assume that $\rho$ is independent of pressure, $d\rho/dp = 0$.

Density varies with temperature, $T$, and with salinity $S$,

$$\rho = \rho(T, S) ,$$

in the manner plotted in fig.(8.4). Here $S$ is defined as the total mass mixing ratio of dissolved salts (about 85% of which is sodium and chloride), usually expressed as g kg$^{-1}$ or, equivalently, o/oo. Temperature $T$ is in $^\circ$C. What is actually plotted in fig.(8.4) is the density anomaly $\sigma$,

$$\sigma = \rho - \rho_o ,$$

(8.1)

the difference between density and a reference value $\rho_0 = 1000$ kg m$^{-3}$. We see that:

1. freshwater is less dense than brackish water; warm water is (almost always) less dense than cold water
2. fresh water ($S = 0$) has a maximum density at about 4°C (which is why ice forms on the top of lakes)

3. in the (rather narrow) range of salinities found in the open ocean (33-36°/oo, see below), $\rho$ is typically 1026 kg m$^{-3}$ and varies monotonically with temperature.

### 8.1.2 Temperature and salinity structure

The distribution of sea surface temperature is shown in Fig.(8.5). Not surprisingly, temperatures are warmest in the tropics (up to almost 30°C) and coldest (0°C) in high latitudes. (Water has an albedo of around 10% - see table 2.2 - depending on surface conditions, and so absorbs solar radiation very efficiently.) Note the strong gradients of SST across the Southern Ocean, and in the western Pacific and North Atlantic Oceans. Note also the E-W temperature gradient across the equatorial Pacific Ocean, with relatively cool temperatures in the east, and the warmest temperatures anywhere west of the Date Line. This latter region coincides with the location of greatest atmospheric convection (cf. fig.4.18), an indication of the importance to climate of the ocean structure.

The coldest waters correspond to regions of deepest ocean convection where the surface ocean communicates with the deep interior (see below and chapter 10). These sites - in the northern north Atlantic Ocean and around Antarctica - play a central role in the thermohaline circulation of the ocean, as discussed in Chapter 10.

Observed mean distributions of $T$, $S$, and $\sigma$ are shown$^2$ in Figs.(8.6), (8.7) and (8.8).

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$^2$In fact, what are shown are potential temperature and potential density, allowing for compressibility.
Figure 8.5: Average (a) DJF and (b) JJA sea-surface temperature (°C).
Figure 8.6: Annual-mean cross-section of zonal-average potential temperature ($^\circ$C) in the world’s oceans: top shows upper km; bottom from 1km to the bottom. Light shading represents warm fluid.
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Figure 8.7: Annual-mean cross-section of zonal-average salinity (‰) in the world’s oceans: top shows upper km, bottom from 1km to the bottom. Lightly shaded fluid is salty.
Figure 8.8: Annual-mean cross-section of zonal average potential density anomaly for the world oceans (referenced to the surface in $kg m^{-3}$) in the top km.

Note that the regions above and below 1000m are shown on different scales. The reason for this is that the ocean structure is different in the two regions. In the abyss, vertical gradients are weak and horizontal gradients are almost nonexistent; e.g., the deep ocean is everywhere very cold (between 0 and 1°C) and no more that 1°C warmer in the tropics than in high latitudes. In the upper kilometer of the ocean - see top panel of figs.(8.6), (8.7) and (8.8) - there are strong vertical gradients (especially of temperature and density); this is the thermocline of the world’s oceans having a depth of $\sim 600m$ in middle latitudes but shallowing to 100 – 200m in low latitudes. The temperature contrast between high and low latitudes is not surprising; the salinity contrast is a little more subtle. In low latitudes, evaporation from the surface is vigorous and exceeds precipitation; since evaporation removes water, but not salt, the near-surface salinity is concentrated. In high latitudes, there is an excess of precipitation over evaporation, and so the surface waters are relatively fresh. Notice how this makes the surface waters less dense than the deep water, despite the fact that at high latitudes the surface waters can be somewhat colder than lower down.
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8.1.3 The mixed layer and thermocline

At the surface of the ocean there is a well-defined *mixed layer* in direct contact with the overlying atmosphere in which properties are relatively uniform in the vertical. The mixed layer depth varies with latitude and season, but is typically 50 to 100m deep - see fig.(8.10). Over the bulk of the ocean the mixed layer communicates with the underlying thermocline, except in high latitudes (particularly in the northern North Atlantic and around Antarctica) where it can get very deep (>1km) and come in to direct contact with the abyss - see chapter 10.
The processes forming the mixed layer are illustrated schematically in Fig. (8.9). Radiation entering the ocean surface is absorbed mostly in the top few meters (depending on wavelength; IR within a few mm, blue light may penetrate to almost 100 m in especially clear water, but usually much less than this). Heat loss, through IR radiation, sensible heat loss to the atmosphere, and evaporation, occur at or within a few mm of the surface. The cooling and salinization of the surface water increases its density. Since, in an incompressible fluid, our criterion for the onset of convection is $\partial \rho / \partial z > 0$, the surface cooling drives convective motions, which stir the mixed layer and tend to homogenize its temperature and other properties, just as in GFD labII. These turbulent motions within the mixed layer may entrain cold water upward across the mixed layer base. In addition, wind stress at the surface drives turbulent motions within the mixed layer, which have the same effect. Furthermore because the base of the mixed layer slopes - see Fig. (8.10) - horizontal currents can carry properties to and from the mixed layer, in a process known as ‘subduction’.

Beneath the mixed layer is the thermocline, the region of rapid changes in $T$ (and $S$) in which the density of the upper waters increases to match those of the abyss. Since density increases (downward) sharply across the thermocline, it is very stable, rather like the stratosphere. Fig. (8.11) shows a detailed hydrographic section along (nominally, along 30°W) in the Atlantic. The structure of the thermocline is clearly evident - contrast this with the highly smoothed, zonally-averaged view of the thermocline evident in Fig. (8.6).

An important distinction between the upper ocean and the abyss is that the former is able to respond reasonably rapidly (days, weeks, years, the time-scale increasing as we go down) to changes in meteorological forcing. However, the deep ocean, by virtue of its very slow circulation and its enormous heat capacity, can only respond very slowly to changing boundary conditions. Thus, for example, the mixed layer (and the upper thermocline) shows seasonal and interannual variations, while the deep ocean only evolves on decadal to centennial (and longer) timescales.
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Figure 8.10: Mixed layer depth (in m) in (top) DJF (Northern hemisphere winter) and (bottom) JJA (Southern hemisphere winter).
8.2 The observed circulation

The observed surface circulation of the ocean is shown in Fig.8.12. This figure shows the general sense of circulation of the surface ocean; as we shall see, a “snapshot” of the currents shows a rich field of eddies superimposed on this average pattern.

Despite the complexity of Fig.8.12, there are a few dominating features. In the “open-ended” Southern Ocean, there is the ‘Antarctic Circumpolar Current’. Elsewhere there are closed circulation patterns called ‘gyres’ because zonal flow is blocked by north-south coasts. In middle latitudes the gyres are anticyclonic, with eastward flow in middle latitudes and westward flow in the tropics. At the western edge of these gyres, there are strong poleward currents, of which the best known is the Gulf Stream in the N Atlantic Ocean, but there are others: the Kuroshio in the N Pacific, and the Brazil current in the S Atlantic. These currents move along the coasts from the tropics, separate from the coasts at about 40° latitude, and then spread eastward across the ocean basin. If we compare fig.(8.12) with fig.(8.5), we can see that these currents are evident in the thermal structure at the ocean surface, most obviously in the strong temperature gradients near the western boundaries of the oceans. By transporting warm water up the western boundaries and then eastward across the oceans, these currents play a role in warming the climate of the eastern oceans and adjacent continents (cf. the temperate climates of Vancouver and Paris, both near 49°N, with the much harsher climate of Newfoundland, at the same latitude in part because of ocean circulation).

The maximum velocities in gyres are found in the western boundary currents, where they reach about 2ms⁻¹. Elsewhere, the currents are substantially weaker. Since the N-S extent of gyres is typically about 20° latitude \( \approx 2000 \text{km} \) (the E-W scale is greater) the maximum Rossby number we can estimate for large-scale gyres is

\[
R_{\text{max}} \sim \frac{U_{\text{max}}}{fL_{\text{min}}} \sim \frac{2 \text{ms}^{-1}}{(10^{-4} \text{s}^{-1}) (2 \times 10^6 \text{m})} \sim 0.01 .
\]

So the Rossby number for large-scale mean motions in the ocean is much smaller than is typical in the atmosphere, and so the geostrophic approximation should be a very good approximation here\(^3\).

\(^3\)This estimate is applicable to the gyre as a whole. Within the western boundary
Figure 8.11: Hydrographic section along 30°W showing the thermal structure in the upper km of the ocean. The thermocline is clearly visible, deep in the subtropics (±30° around the equator), shallow in equatorial regions.
Figure 8.12: Major surface currents of the world oceans.
The mean circulation in the abyss is very slow, except in regions of western boundary currents and where flow is channeled by topography. There is significant stirring of the abyssal ocean by baroclinic eddies, however.

8.3 Inferences from geostrophic and hydrostatic balance

Water obeys the same fluid dynamics as air, so we have already derived the equations we will need. Specifically, our equations of motion on the sphere given by eq.6.5.2 and 6.5.3 of chapter 6. One simplification we can make in application of these equations to the ocean is to recognize that the density varies rather little in the ocean (by no more than about 3%), so we can rewrite the horizontal momentum equations 6.5.3a,b thus (using a local Cartesian coordinate system):

\[
\frac{Du}{Dt} - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \mathcal{F}_x ;
\]

\[
\frac{Dv}{Dt} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \mathcal{F}_y ;
\]

(8.2)

without incurring serious error, where \(\rho_0\) is a constant reference density, 1000 kg m\(^{-3}\), say. But we cannot sensibly do the same thing in the vertical equation, 6.5.3c (hydrostatic balance) which we write in terms of the density anomaly, eq(8.1):

\[
\frac{\partial p}{\partial z} = -g(\rho_0 + \sigma).
\]

If we neglect the contribution from \(\sigma\), we would conclude (assuming \(p \rightarrow 10^5\) Pa - one atmosphere - at the flat surface \(z = 0\)) that pressure is a function of depth only, and thus that there are no horizontal pressure gradients. This is clearly not the case, and we expect, as in the atmosphere, that horizontal gradients of density and pressure are very important in the oceanic circulation.

currents, a more relevant estimate is \(U/f\Delta\), where \(\Delta\) is the width of the current itself. This width is much less than 2000 km, so the characteristic Rossby number within these boundary currents is much greater than the number we have deduced for the gyres. Here the Rossby number can approach unity.
We have seen that ocean currents are much slower than atmospheric winds, so the Rossby number is typically much smaller in the ocean than in the atmosphere, so the geostrophic equations are valid in most circumstances of interest to us. From (8.2), the geostrophic relation can be written

\[
    u = -\frac{1}{f \rho_o} \frac{\partial \rho}{\partial y},
\]
\[
    v = \frac{1}{f \rho_o} \frac{\partial \rho}{\partial x}.
\]  
(8.3)

Taking the vertical derivative, and using hydrostatic balance, gives us our thermal wind equation for the ocean, whose components are:

\[
    \frac{\partial u}{\partial z} = -\frac{g}{f \rho_o} \frac{\partial \sigma}{\partial y},
\]
\[
    \frac{\partial v}{\partial z} = \frac{g}{f \rho_o} \frac{\partial \sigma}{\partial x}.
\]  
(8.4)

where, of course, the density anomaly depends on both the temperature and the salinity distribution.

Using eq(8.4) one can immediately infer the sense of the thermal wind shear from the \( \sigma \) field shown in, for example, fig.(8.8): \( \frac{\partial u}{\partial z} > 0 \) where \( \sigma \) increases moving northward, implying that \( u \) is directed eastward in these regions if abyssal currents are weak. Inspection of fig.(8.8) suggests that \( \frac{\partial u}{\partial z} > 0 \) poleward of 30°N, more or less as observed in fig.(8.12).

**8.3.1 Ocean surface structure and geostrophic flow**

**Near-surface geostrophic flow**

To the extent that the surface currents plotted in fig.(8.12) are in geostrophic balance, there must be a pressure gradient force balancing Coriolis forces acting on them. Consider, for example, the eastward flowing Gulf Stream of the Atlantic. In order to balance southward directed Coriolis forces acting on it, there must be a pressure gradient force directed northward. This is provided by a tilt in the free surface of the ocean: we shall see that the sea surface is higher (relative to the geoid) in the subtropics than further to the north. Consider fig.(8.13). If we consider some horizontal surface of constant
8.3. INFERENCE FROM GEOSTROPHIC AND HYDROSTATIC BALANCE

Figure 8.13:

$z$, then we can integrate the hydrostatic relation up to the free surface (where $p = p_s$, atmospheric pressure) to give:

$$p(z) = p_s + \int_z^h \rho_g \, dz = p_s + g \langle \rho \rangle (h - z) ,$$

where $\langle \rho \rangle = \frac{1}{(h-z)} \int_z^h \rho \, dz$ is the mean density in the column of depth $h - z$.

If we are interested in the near-surface region ($z = z_0$, say, in the figure), fractional variations in column depth are much greater than those of density, so we can neglect the latter, setting $\rho = \rho_0$ and leaving

$$p(z_0) = p_s + g \rho_0 (h - z_0) .$$

Horizontal variations in pressure in the near-surface region will thus depend on variations in atmospheric pressure and in free-surface height. Since here we are interested in the long time scale of the ocean circulation, we can neglect day-to-day variations of atmospheric pressure to conclude that the horizontal components of the near-surface pressure gradient are given by gradients in surface elevation

$$\mathbf{\nabla} \times \hat{z} \times \nabla p = g \rho_0 \hat{z} \times \nabla h .$$

Thus, the geostrophic flow just beneath the surface is

$$\mathbf{u}_{surface} = \frac{1}{f \rho_0} \hat{z} \times \nabla p$$
Note how (8.6) exactly parallels the equivalent relationship (6.57) for geostrophic flow on an atmospheric pressure surface. Maps of the height of the sea surface, \( h \), give us the same information as do maps of the height of atmospheric pressure surfaces (and, of course, the ocean surface is — to a very good approximation — a surface of constant pressure). In chapter 6.6 we saw that geostrophic winds of \( 10\, \text{ms}^{-1} \) were associated with tilts of pressure surfaces by \( \sim 500\, \text{m} \) over a distance of \( 5000\, \text{km} \). But because oceanic flow is weaker than atmospheric flow, we expect to see much gentler tilts of pressure surfaces in the ocean. We can estimate the size of expected \( h \) variations by making use of eq.(8.6) along with observations of surface currents: if \( U \) is the eastward speed of the surface current, then \( h \) must drop by an amount \( \Delta h \) in a distance \( L \) given by:

\[
\Delta h = \frac{fLU}{g}
\]

or \( 1\, \text{m} \) in \( 1000\, \text{km} \) if \( U = 10^{-1}\, \text{ms}^{-1} \) and \( f = 10^{-4}\, \text{s}^{-1} \).

**Observations of surface elevation**

If we could observe the \( h \) field of the ocean then, just as in the use of geopotential height maps in synoptic meteorology, we could deduce the surface geostrophic flow in the ocean. Amazingly variations in ocean topography, \( h \), even though only a few cms to a metre in magnitude, can indeed be measured from satellite altimeters and are mapped routinely over the globe every week or so. Flying at a height of \( \sim 1000\, \text{km} \) above the earth’s surface, altimeters measure their height above the sea surface to a precision of a \( \text{cm} \) or two. And, tracked by lazers, their distance from the center of the earth can also be determined to high accuracy, permitting \( h \) to be found by subtraction.

The annually-averaged surface elevation (relative to the mean “geoid”, the shape the ocean’s surface would take up if it were not moving) is shown in fig.(8.14). As we expect, consistent with fig.(8.12), the highest elevations are in the anticyclonic subtropical gyres (where the surface is about 40cm higher than near the eastern boundaries at the same latitudes), and there are strong gradients of height at the western boundary currents and near the circumpolar current of the Southern Ocean (surface height changes by about 1m across these currents).
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Figure 8.14: The mean height of the sea surface relative to the geoid (in cms) as measured by the Topex-Poseidon satellite altimeter. The pressure gradient force associated with the tilted free surface is balanced by Coriolis forces acting on the geostrophic flow of the ocean at the surface.

8.3.2 Deep geostrophic flow

At depths much greater than variations of \( h \) (at \( z = z_1 \), say, in Fig.8.13), we can no longer neglect variations of density in (8.5) in comparison with those of column depth. Again neglecting atmospheric pressure variations, horizontal pressure variations at depth are therefore given by

\[
\mathbf{z} \times \nabla p = g \langle \rho \rangle \mathbf{z} \times \nabla h + g(h - z) \mathbf{z} \times \nabla \langle \rho \rangle ,
\]

and therefore the deep water geostrophic flow is given by

\[
\mathbf{u} = \frac{1}{f \rho_0} \mathbf{z} \times \nabla p = \frac{g}{f \rho_0} \left[ \langle \rho \rangle \mathbf{z} \times \nabla h + (h - z) \mathbf{z} \times \nabla \langle \rho \rangle \right] \approx \frac{g}{f} \mathbf{z} \times \nabla h + \frac{g(h - z)}{f \rho_0} \mathbf{z} \times \nabla \langle \rho \rangle , \tag{8.7}
\]

since we can approximate \( \langle \rho \rangle \approx \rho_0 \) in the first term. This has two contributions: that associated with free-surface height variations, and that associated
with ocean density gradients. Note that if the ocean density is uniform, the second term vanishes and the deep-water geostrophic flow is the same as that at the surface: \textit{geostrophic flow in an ocean of uniform density is independent of depth}. This is of course a manifestation of the Taylor-Proudman theorem.

The second term in eq(8.7) is the “thermal wind” term, telling us that $u$ will vary with depth if there are horizontal gradients of density. Thus, there may be a nonzero flow at depth even if the surface is flat. Conversely, the presence of horizontal variations in surface height, manifested in surface geostrophic currents as in eq(8.6), does not guarantee a geostrophic flow at depth. Indeed, typically the flow becomes much weaker at depth—the Taylor columns do not extend into the deep ocean. This must mean that the density field adjusts in such a way that the second term in eq(8.7) cancels the first.

Thus in the subtropical gyre (where the sea surface is high - see fig.(8.14)) we observe isotherms bowing down in to the interior of the ocean. In the sub-polar gyre (where the sea surface is depressed) we observe isotherms bowing up toward the surface - see fig.(8.11). It is these horizontal density gradients interior to the ocean that, through the hydrostatic equation, ‘buffer out’ horizontal gradients in pressure as we move down the water column - the second term in eq.(8.7). This is illustrated schematically in the fig.(8.15).